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Global and Local Spin Squeezing in Coupled Quantum Kicked Tops Model

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Abstract We investigate the global and the local spin squeezing in a weakly coupled quantum kicked tops system. Two different situations are considered: (i) $N = 1$ and (ii) $N = 30$ for each subsystem, corresponding to quantum and classical cases, respectively. In the first case, since the two subsystems have no spin squeezing, the global squeezing completely originates from quantum correlations. For the second one, the global spin squeezing is enhanced over the local one. Due to the chaotic nature of the system, the spin squeezing is sensitive to the initial state. In chaotic region, the squeezing vanished time is much shorter than that in the regular region.

Keywords Spin squeezing · Quantum correlation · Quantum chaos

1 Introduction

Due to possible applications in high-precision atomic clocks [1–4] and quantum information [5–11], spin squeezing in collective spin system has received much attention for decades [12–14]. Also, spin squeezing is a purely quantum effect that leads to fundamental physical studies [1–3, 15, 16]. By now, it is found that spin squeezing is closely related to quantum entanglement [5, 9, 17], which plays an key role in quantum information and computation.

The concept of spin squeezing has been clarified by Kitagawa and Ueda [1], which associates both quantum correlations and indeterminacy relationships. Therefore, spin squeezing

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of collective spin system means reducing spin fluctuation in one certain direction perpendicular to the mean spin. An ensemble of N spin-s particles can be described by the following collective operators [18],

$$\hat{J}_\alpha = \sum_{k=1}^{N} \hat{S}_\alpha^{(k)} \quad (\alpha = x, y, z), \tag{1}$$

where $\hat{S}_\alpha^{(k)}$ are the angular momentum operators for the kth spin, with magnitude s. The spin squeezing is quantified by the following parameter [1]

$$\xi^2 = \frac{2(\Delta \hat{J}_{\mathbf{n}_\perp})^2}{J}, \tag{2}$$

where the subscript $\mathbf{n}_\perp$ refers to an axis perpendicular to the mean spin direction $\mathbf{n} = \langle \hat{\mathbf{J}} \rangle / |\langle \hat{\mathbf{J}} \rangle|$, where the minimal value of the variance $(\Delta \hat{J}_{\mathbf{n}_\perp})^2$ is obtained, $J_{\mathbf{n}_\perp} = \hat{\mathbf{J}} \cdot n_\perp$, and the magnitudes satisfy $J = Ns$. The inequality $\xi^2 < 1$ indicates that the state is spin squeezed and for spin coherent state (SCS), $\xi^2 = 1$.

In Ref. [19], the authors have studied the relations between the global and local spin squeezing in a composite system, and found that for a pure symmetric product state, the global squeezing parameter is equal to the local one. And this can be treated as a criterion for entanglement.

In this paper, we study the global and local spin squeezing in a weakly coupled quantum kicked tops (CQKTs) model, which is an important model in studying quantum correlation and quantum chaos [20–25]. For an arbitrary symmetric system, which consists of two identical subsystems, we prove explicitly that the mean spin direction (MSD) of the global system is always equal to any subsystem in the dynamical evolvement when initial state is symmetrical. In our paper, two different situations are considered: (i) $N = 1$ and (ii) $N = 30$ for each subsystem. For the first condition, the global spin squeezing completely originates from quantum correlations and for the second one, the global spin squeezing is enhanced whatever the initial coherent state is located in regular or chaotic regions. However, the underlying chaotic motion greatly affects the spin squeezing vanished time, namely, spin squeezing vanishes after a very short time for an initial coherent centered in a chaotic region of the phase space, whereas the spin squeezing occurs over a longer time for the coherent centered on a regular region.

2 Coupled Kicked Tops

2.1 Quantum Top

The CQKTs are governed by the Hamiltonian ($\hbar = 1$ hereafter) [20]

$$\hat{\mathcal{H}}(t) = \hat{H}_1(t) + \hat{H}_2(t) + \hat{H}_{12}(t), \tag{3}$$

where

$$\hat{H}_1(t) = p_1 \hat{J}_{y_1} + \frac{\kappa_1}{2j_1} \hat{J}_{z_1}^2 \sum_n \delta(t-n),$$

$$\hat{H}_2(t) = p_2 \hat{J}_{y_1} + \frac{\kappa_2}{2j_2} \hat{J}_{z_1}^2 \sum_n \delta(t-n), \tag{4}$$

$$\hat{H}_{12}(t) = \epsilon\sqrt{\frac{\kappa_1\kappa_2}{j_1 j_2}}\hat{J}_{z_1}\hat{J}_{z_2}\sum_n \delta(t-n).$$

Here, $(J_{x_\mu}, J_{y_\mu}, J_{z_\mu})$ $(\mu = 1, 2)$ are angular momentum operators for the μth spin, they obey the commutation relation $[J_{x_\mu}, J_{y_\mu}] = iJ_{z_\mu}$. p_μ are the strengths for each kick and the J_{y_μ} terms are manifested as turns around y-axis of each top. κ_μ are the strengths of the twists and the $\hat{J}^2_{z_\mu}\sum_n \delta(t-n)$ terms are periodic δ-function kicks. Therefore, the Hamiltonian is an alternative sequence of twists ($J^2_{z_\mu}$ terms) and turns (J_{y_μ} terms). $\hat{H}_1(t)$ and $\hat{H}_2(t)$ are the Hamiltonians for each individual tops, respectively, and $\hat{H}_{12}(t)$ is the coupling between the tops, described by spin-spin interaction term with a coupling strength of $\epsilon\sqrt{\kappa_1\kappa_2/j_1 j_2}$. If $\epsilon = 0$, we can obtain two noninteracting single QKTs.

The dynamical description of the CQKTs is via the Floquet operator

$$\hat{F} = \hat{F}^\epsilon_{12}(\hat{F}_1 \otimes \hat{F}_2) = \hat{F}^\epsilon_{12}\left[(\hat{F}^k_1\hat{F}^f_1 \otimes (\hat{F}^k_2\hat{F}^f_2)\right], \tag{5}$$

where the different terms are given by,

$$\begin{aligned} &\hat{F}^f_\mu \equiv \exp\big(-ip_\mu\hat{J}_{y_\mu}\big), \qquad \hat{F}^k_\mu \equiv \exp\left(-i\frac{\kappa_\mu}{2j_\mu}\hat{J}^2_{z_\mu}\right), \\ &\hat{F}^\epsilon_{12} \equiv \exp\left(-i\epsilon\sqrt{\frac{\kappa_1\kappa_2}{j_1 j_2}}\hat{J}_{z_1}\hat{J}_{z_2}\right). \end{aligned} \tag{6}$$

Here we set the strength of "twists" κ the same in two tops, and in the following we set $p = \pi/2$, $j_1 = j_2$ and vary κ. In the Schödinger picture, an arbitrary initial state $|\psi(0)\rangle$ evolves to

$$|\psi(n)\rangle = \hat{F}^n\,|\psi(0)\rangle\,. \tag{7}$$

2.2 Classical Top

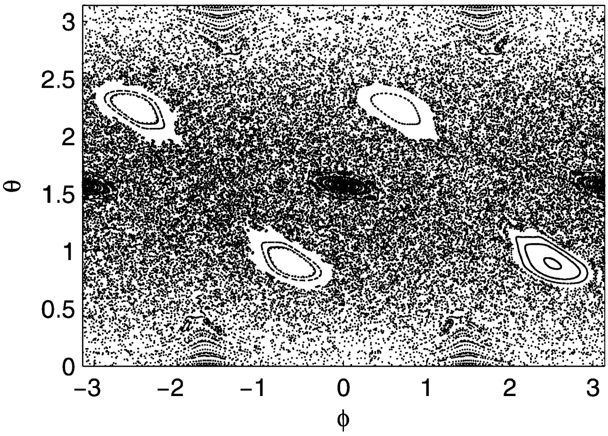
The classical limit of the CQKTs is obtained by expressing $X_\mu = \langle\hat{J}_{x_\mu}\rangle/j$ and factorizing all moments such as $X_\mu Y_\mu = \langle\hat{J}_x\hat{J}_y\rangle/j^2$ to products of first-order moments. To see this, the rescaled angular momentum commutation operators will commute and become c-number variables in the limit of $j \to \infty$. Then the classical equations of motion corresponding to Hamiltonian (3) are given by

$$\begin{aligned} X'_1 &= Z_1\cos\Omega_{12} + Y_1\sin\Omega_{12}, \\ Y'_1 &= -Z_1\sin\Omega_{12} + Y_1\cos\Omega_{12}, \\ Z'_1 &= -X_1, \\ X'_2 &= Z_2\cos\Omega_{21} + Y_2\sin\Omega_{21}, \\ Y'_2 &= -Z_2\sin\Omega_{21} + Y_2\cos\Omega_{21}, \\ Z'_2 &= -X_2 \end{aligned} \tag{8}$$

where

$$\Omega_{12} \equiv \kappa X_1 + \epsilon X_2 \quad \text{and} \quad \Omega_{21} \equiv \kappa X_2 + \epsilon X_1. \tag{9}$$

Fig. 1 Stroboscopic phase-space pictures of the classical single kicked top (a) $\kappa = 3$ and the coupled kicked tops (b) $\kappa = 3$, $\epsilon = 0.05$



As we all know, in the limit $\epsilon \to 0$, we will obtained the single QKT, whose Hamiltonian is given by,

$$\begin{aligned} X' &= Z \cos\kappa X + Y \sin\kappa X, \\ Y' &= -Z \sin\kappa X + Y \cos\kappa X, \\ Z' &= -X. \end{aligned} \tag{10}$$

The dynamics of the single top have been studied extensively in Ref. [26–28]. From the above expressions, it is clear that the stroboscopic evolution can be represented in a phase space given by a sphere S^2 of a unit radius. The classical, normalized angular momentum variables (X, Y, Z) can be parametrized in polar coordinates as $(X, Y, Z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, where θ and ϕ are the polar and azimuthal angles, respectively, and this renders the classical map two dimensional.

To express the classical dynamics of the CQKTs intuitively, we give a stroboscopic phase-space picture for the classical map shown in Fig. 1. In the plot, we choose the chaotic parameter $\kappa = 3$, which yields a mixture of regular and chaotic areas of a significant size. Elliptic fixed points surrounded by the chaotic sea are evident (see Fig. 1), and they have coordinates $(\theta, \phi) = (2.25, 0.63)$ and $(\theta, \phi) = (0.89, 0.63)$.

3 Global and Local Squeezing

The QKT, as a representative spin model, exhibits chaotic behaviors in the classical limit [29], Song et. al. found that the classical chaos suppresses quantum squeezing. Recently, there are some approaches to investigate the quantum correlation for the CQKTs model [21–23, 25]. The quantum correlation plays an important role in the study of the global and local squeezing [19]. Inspired by these, we choose CQKTs as a composite system to investigate the spin squeezing.

In this paper, the initial state is a pure separable state

$$|\Psi(0)\rangle = |\theta_0, \psi_0\rangle \otimes |\theta_0, \psi_0\rangle\,, \tag{11}$$

where $|\theta_0, \psi_0\rangle = \exp\{i\theta_0[\hat{J}_x \sin\phi_0 - \hat{J}_y \cos\phi_0]\}|j, j\rangle (0 \le \theta_0 \le \pi, -\pi \le \phi_0 \le \pi)$ is a SCS. The angular momentum Hilbert space $|j, j\rangle$ is $(2j + 1)$-dimensional and we will work in a $[(2j+1)\times(2j+1)]$-dimensional product space. To study spin squeezing, a SCS is the most appropriate initial state and the SCS, whose centre is on a classical phase-space point, can build the connection between the quantum and classical dynamics of QKT.

3.1 Mean Spin Directions

According to the definitions of spin squeezing, we first need to determine the MSD. Here, we study relations between the global and local MSDs for any symmetric two-spin systems

$$[\hat{P}_{12}, \hat{\mathcal{H}}] = 0, \tag{12}$$

where $\hat{P}_{12}$ is an exchange operator. Supposing a pure symmetric product state as our initial state

$$|\Psi\rangle = |\psi\rangle \otimes |\psi\rangle, \tag{13}$$

by considering the exchange symmetry, the global MSD is equal to the local one

$$\mathbf{n}_g(t) = \mathbf{n}_l(t). \tag{14}$$

Therefore, the relation between the global and local spin squeezing is easily obtained as

$$\xi_g^2 = \xi_l^2 + \frac{4\mathrm{cov}(\hat{S}^{(1)}_{n_\perp}, \hat{S}^{(2)}_{n_\perp})}{j}, \tag{15}$$

where $\mathrm{cov}(\hat{A},\hat{B}) = \langle\hat{A}\hat{B}\rangle - \langle\hat{A}\rangle\langle\hat{B}\rangle$ is the covariance for $\hat{A}$ and $\hat{B}$. Thus we can see that, the difference between the global and local squeezing is from the covariance term, which describes the correlation between the two subsystems. If the covariance is zero, there is no correlations, and thus the subsystem is independent, the global squeezing is just equal to the local one.

The mean spin direction $\mathbf{n}_1$ is determined by the expectation values $\hat{J}_\alpha (\alpha = x, y, z)$, and the other two directions $\mathbf{n}_2$ and $\mathbf{n}_3$ are normal to $\mathbf{n}_1$. In the spherical coordinate,

$$\begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n}_3 \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\sin\phi & \cos\phi & 0 \\ -\cos\theta\cos\phi & -\cos\theta\sin\phi & \sin\theta \end{bmatrix} \tag{16}$$

where θ and ϕ are the polar and azimuthal angles, respectively. The angles are given by $\theta = \arccos(\langle\hat{J}_z\rangle/|\hat{\mathbf{J}}|)$, $\phi = \arctan(\langle\hat{J}_y\rangle/\langle\hat{J}_x\rangle)$.

From (3), we know the CQKTs is a symmetric system, and we employ a pure and symmetric directed product state $|\Psi(0)\rangle = |\theta_0, \psi_0\rangle \otimes |\theta_0, \psi_0\rangle$ as an initial state, where $(\theta_0, \psi_0) = (2.25, 0.63)$. The numerical results of the global MSDs and the local ones are given in Fig. 2. As we expected, the global (circle points) and local angles always coincide with each other in the dynamical evolutions.

3.2 Spin Squeezing

Secondly, in order to obtain the spin squeezing parameter ξ^2, we need to compute the following minimal variance

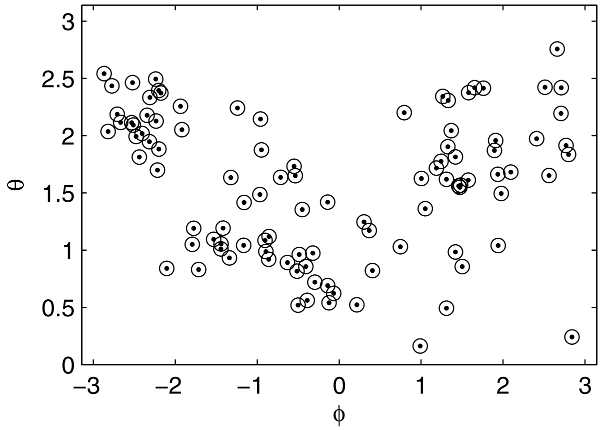


Fig. 2 Distributions of the global mean spin directions (*circle points*) and the local ones (*dot points*) for initial CCSs for each top with $\theta_0 = 2.25$ and $\phi_0 = 0.63$. The number of kicks for each subplot is 100

$$(\Delta J_{\mathbf{n}_\perp})^2 = \frac{1}{2}\langle \Delta J_{\mathbf{n}_2}^2 + \Delta J_{\mathbf{n}_3}^2 \rangle - \frac{1}{2}\sqrt{\langle \Delta J_{\mathbf{n}_2}^2 - \Delta J_{\mathbf{n}_3}^2 \rangle^2 + \langle [J_{\mathbf{n}_2}, J_{\mathbf{n}_3}]_+ \rangle^2}, \tag{17}$$

with

$$\begin{aligned} J_{\mathbf{n}_2} &= -J_x \sin\phi + J_y \cos\phi \\ J_{\mathbf{n}_3} &= -J_x \cos\theta \cos\phi - J_y \cos\theta \sin\phi + J_z \sin\theta, \end{aligned} \tag{18}$$

where $[\hat{A}, \hat{B}]_+ = \hat{A}\hat{B} + \hat{B}\hat{A}$. Then, we focus on the dynamics of the squeezing for the global and local system.

In the case of $N = 1$, the system consists of two qubit, the Floquet operator can be written explicitly. In principle, the analytical results for spin squeezing parameter can be obtained for arbitrary initial states, however, the MSDs are very difficult to be determined except for some special initial states. Therefore, to illustrate the dynamic of the squeezing parameter, without loss of generality, we choose the initial state as $|0, 0\rangle \otimes |0, 0\rangle$, which is also easy to be prepared in practice by just cooling the two qubits to their ground states. The expectation values for the total angular momentum are

$$\begin{aligned} \langle J_x \rangle &= \frac{1}{2}(\cos n\pi - 1)\cos\frac{(n-1)\epsilon\kappa}{2}, \qquad \langle J_y \rangle = 0, \\ \langle J_z \rangle &= -\frac{1}{2}(1 + \cos n\pi)\cos\frac{n(\pi + \epsilon\kappa)}{2}, \end{aligned} \tag{19}$$

thus the MSD is in $-z$ direction for the initial state ($n = 0$), and then it points to x, z, $-x$ directions periodically with respect to the external pulses. We obtain the exact spin squeezing parameter,

$$\xi_g^2 = \begin{cases} 1 - |\sin(n\epsilon\kappa/2))|, & n \text{ is even}, \\ 1 - |\sin[(n+1)\epsilon\kappa/2]|, & n \text{ is odd}. \end{cases} \tag{20}$$

In Fig. 3(a), we plot the global and local spin squeezing parameters for weak coupling strength $\epsilon = 0.01$. According to the notion of the spin squeezing, one spin-1/2 has no spin

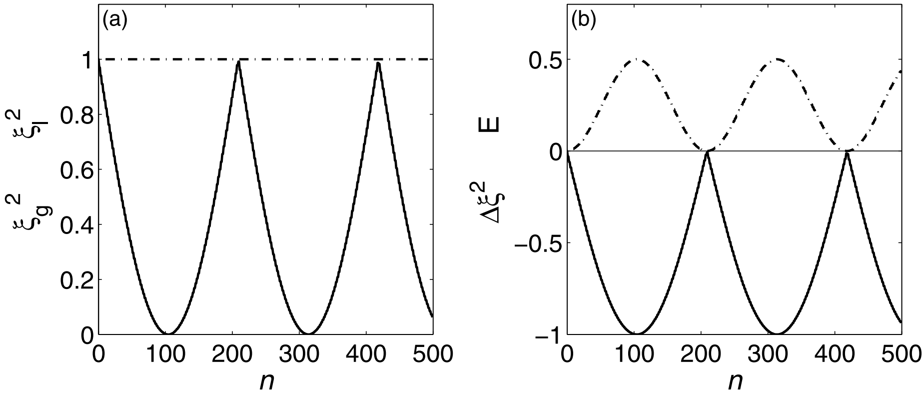


Fig. 3 Dynamical evolutions of the global (*solid line*) and local (*dashed dotted line*) spin squeezing parameter (**a**), the linear entropy (*dashed dotted line*) and $\Delta\xi^2$ (*solid line*) versus time n (**b**). Each top with initial state SCS $(\theta_0, \phi_0) = (0, 0)$, the other parameters are $\kappa = 3$ and $N = 1$

squeezing, i.e., $\xi_l^2 = 1$. The global squeezing oscillates as the above equation with period $2\pi/\epsilon\kappa$, when $\xi_g^2 = 1$, the global squeezing vanishes. Note that, if we choose the specific value of κ in (20), there are two cases for $\xi_g^2 = 0$. In the case n is even, the state degenerates to a GHZ state $|\psi\rangle = a|0,0\rangle\langle 0,0| + b|1,1\rangle\langle 1,1|$, with $\{|0\rangle, |1\rangle\}$ the eigenstates of σ_z, while if n is odd, the state $|\psi\rangle = a|0,0\rangle\langle 0,0| + b|1,1\rangle\langle 1,1|$, with $\{|0\rangle, |1\rangle\}$ the eigenstates of σ_x. Actually, there is no proper definition for spin squeezing since there is no MSD for a GHZ state. The linear entropy can also be obtained as

$$E = \begin{cases} [1 - \cos(n\epsilon\kappa)]/4, & n \text{ is even}, \\ \{1 - \cos[(n+1)\epsilon\kappa]\}/4, & n \text{ is odd}. \end{cases} \tag{21}$$

In Fig. 3(b), we plot the linear entropy and the difference between the squeezing parameters, i.e., $\Delta\xi^2 = \xi_g^2 - \xi_l^2$, it can be seen that if $\Delta\xi^2 = 0$, there is no entanglement, and if $\Delta\xi^2 \neq 0$, the entanglement is present. Since the two subsystems has no spin squeezing, the global squeezing completely originates from quantum correlations [19].

As the results given in [29], quantum chaos suppresses spin squeezing strongly, the authors have found that it is convenient to introduce the spin squeezing vanished time t_v, after which the system is no longer spin squeezed, to characterize quantum chaos. The correspondence of the quantum and classical dynamics of QKT is done only if the number of particles is large enough [29]. As a composite system, the CQKTs could be treated classically when $N = 30$ for each top. In Fig. 4, we choose SCS centred in the regular region (see Fig. 1) as the initial state for each top and the interaction is weakly coupled. We find that the global optimal squeezing (minimal squeezing parameter) is better than the local one in the early time. In this sense, the enhanced global squeezing of a CQKTs is similar to the one of a single QKT as the initial state centred in a more regular region. And it is noted that, the global spin squeezing parameter is more than the local one sometimes as the dynamics takes place. These results are from positive or negative covariance between two spins [19]. As the initial SCS centred in the chaotic region (see Fig. 4(b)), we also observe that the global squeezing is strengthened, whereas spin squeezing vanished time t_v becomes very short. The difference between the local and global spin squeezing is interesting, when the CQKTs just consists of two qubits, the difference is very obvious, since a single qubit is never squeezed, the global squeezing comes completely from the covariance. When the system size increases,

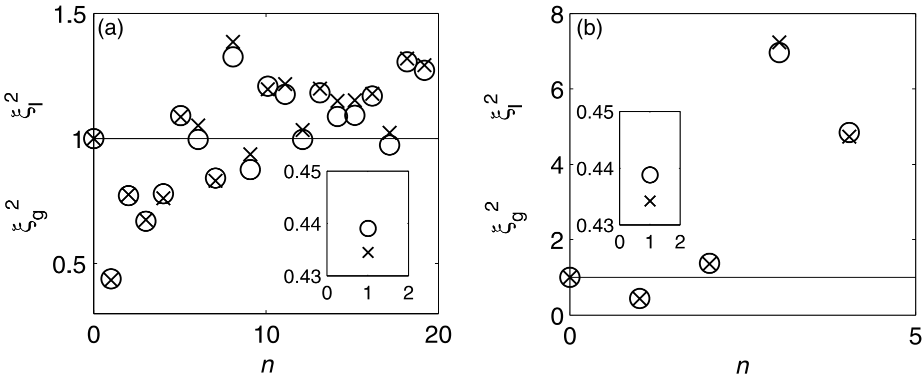


Fig. 4 Dynamical evolution of the global spin squeezing parameter (*cross points*) and the local one (*circle points*) for different initial state: (**a**) Each top with initial state SCS $(\theta_0, \phi_0) = (2.25, 0.63)$; (**b**) each top with initial state SCS $(\theta_0, \phi_0) = (0.89, 0.63)$. The other parameters are $\kappa = 3$ and $N = 30$ for each top, coupling strength $\epsilon = 0.01$

the difference becomes very small, the covariance term is small, and the global squeezing is mainly from the local one.

4 Conclusions

We have studied the global and local spin squeezing in the CQKTs under the quantum and classical conditions. For a two-spin system with exchange symmetry, if the initial state is also symmetric, the global MSDs are equal to the local ones in the dynamic evolution. Then a relation between the global and local squeezing was obtained. For the system consists of two spin-1/2 particles, we derived the spin squeezing parameter analytically. As there is no spin squeezing for a single spin-1/2 particle, the global spin squeezing originates from quantum correlations absolutely. On the other hand, if and only if the difference between the global and local spin squeezing parameters is equal to zero the entanglement exists. For the condition $N = 30$ that the system can be treated as classical, the global spin squeezing is stronger than the local one whatever the initial coherent states are centred in regular or chaotic regions. However, the underlying chaotic motion greatly affects the existence of spin squeezing, namely, the vanished time for an initial coherent centered in a chaotic region is shorter than the one for the coherent centered on a regular region.

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